

# Introduction to Game Theory

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# Game theory

- Game theory (GT) is a theory of strategic decision making
- It applies to all those situation (very common) where individual actions affect competitors' response and viceversa and where individual payoff depends on both his or her own actions and competitors' actions
  - eg. consider a litigation at court

# Game theory

- GT has been applied to study:
  - industrial organization (collusion, price predation, anti-competitive behaviors relevant to antitrust legislation and the action of antitrust authorities)
  - contract theory
    - the relationship between a principal and an agent under asymmetric information
      - how should a contract be written when the principal does not know whether the agent has low or high production costs?
      - or when the principal does not perfectly observe whether the agent is providing enough effort or is shirking?
      - how should the contract be written if agent's obligation is not enforceable by a third party (court or judge) in case of breach, perhaps because agent's action (eg production) cannot be observed or is not verifiable?
      - how should the contract be written if agent is protected by a limited liability which prevents it from being harshly punished in case of breach?
      - ...

# Game theory

- what should be the optimal default clauses (stated by law) for incomplete contracts (i.e. contracts that do not determine what happens when partners obligations are not held, or when particular "states of the world" occur?)
- optimal strategies in litigations
- how should political parties determine their political program in order to win next elections?
- how should the members of a board vote?
- how should governments decide their military spending?
- where should admirals place the fleet to win a major sea battle?

# Game theory

- how should OPEC members decide about their annual output?
- how should AMD and Intel decide how much to invest in R&S?
- optimal marketing campaign strategies
- pharmaceutical companies face decisions regarding whether to market a product immediately and gain a competitive edge over rival firms, or prolong the testing period of the drug

# Game theory

- if a bankrupt company is being liquidated and its assets auctioned off, what is the ideal approach for the auction
- ideal auction tactics
  
- how should you play chess? and poker? and tick tack toe? and any other game?
  
- how psychology affects individual beliefs and belief formation about other competitors' action?

# Game theory

- The basics of the theory were developed in the Forties by a mathematician (Von Neumann) and an economist (Morgenstern)
- Since then GT gained an enormous relevance in many disciplines, mainly economics, law, political science, psychology, physics, biology and even philosophy.
- Most people believe that GT is a branch of mathematics, but it is not true. It is a core part of social sciences that uses mathematical tools, but only as a language. Key concepts (those that we shall discuss) refer to decision making
- GT is a revolutionary interdisciplinary phenomenon bringing together psychology, mathematics, philosophy and an extensive mix of other academic areas.

# Game theory

- GT created a new paradigm for studying human behavior and interaction, quite different from the previous paradigm of the neoclassical microeconomics
- While in microeconomics individuals are always considered as price-takers, that act in isolation and do not care of the influence of their actions on other people, or on the effects of others' actions on themselves, GT explicitly deals with strategic interaction

# Game theory

- Eight Noble Prizes have been awarded to scientists who have progressed the discipline.
- But beyond the academic level, is game theory actually applicable in today's world?
- Yes!
- GT is taught to MBAs and law schools and it is currently applied by big companies to decide their business strategies
  - there is too much at stake to fail: decisions need to be taken as much carefully as possible

# Simple examples – Prisoner's dilemma

The most famous game: the prisoner's dilemma.  
How would you play?

	<b>confess</b>	<b>do not confess</b>
<b>confess</b>	-3, -3	0, -5
<b>do not confess</b>	-5, 0	-1, -1

# Simple examples – Prisoner's dilemma

- Prisoner's dilemma represents a very common situation
  - individually there is an incentive to deviate from cooperation when you expect your opponents cooperate
  - but this behavior leads to the worst possible outcome socially.
- where this situation applies?
  - collusion
  - carbon emission
  - psychology: addiction problem. Today Self vs Tomorrow Self decide whether to drink or not. Drinking today if I expect Tomorrow Self will not drink is a tempting deviation
  - advertising / weapon / R&D race: cooperation would be spend little; deviation is spend a lot when I expect the opponent will spend little.
  - doping in sport

# Simple examples – Main Street

- Location of an Hot Dog Stand (or Restaurant, Store, etc.)
- Suppose there are two hot dog vendors that operate along Main Street
- All along Main Street there are 7 blocks of office buildings, each hosting 10 employees that go for an hot dog at lunchtime.
- Employees dislike walking because their lunch break is short
- To make things simple, suppose that each vendor sells same quality hot dogs
- You (one of the vendors) have to decide where locating your hot dog stand

# Simple examples – Main Street

- The conclusion is to locate your business right next to your competitor
- If you think about it, this is actually what restaurants do in most cities, usually they tend to form clusters
- This framework doesn't just apply to restaurants. Think about political campaigns.
- Notice that the outcome is the production of two products from different competitors that are exactly the same (same distance from the customers' viewpoint). This is what some people have referred to as the “Principle of Minimum Differentiation.”

# Ingredients of a game

- who are the players?
- what can players do?
- when do players move?
- what do players know when they move?
- what are the payoffs associated to each possible outcome of the game

# Hypothesis

- players are rational
  - i.e. they want to maximize their payoff, they want to win
  - therefore they choose the strategy that yields the highest payoff
- common knowledge of the game description
  - all players have the same information regarding game description
  - all players know that their opponents know game description
  - all players know that their opponents know that they know...
- common knowledge of game description does not mean that players have the same information:
  - some player characteristics could be private information

# Games and information

- We will consider games with
  - complete and symmetric information (all players know their opponents' characteristics)
  - incomplete (asymmetric) information
- Games could be simultaneous or dynamic
  - dynamic games could have perfect information (previous history is known to all players at each point in time) or imperfect information (previous history partially unknown)
- Key observation: simultaneous games could be represented as dynamic games with imperfect information

# Goal

- We want to predict the outcome of the game, before the game is actually played
- GT serves to make predictions!
- In the following lectures we shall see a number of criteria, maximally reasonable, to discard possible strategies and to identify the outcome of the game.
- In many cases, by applying these criteria we will find many possible outcomes for a given game, all equally possible.
- The quest will be for further requirements to impose to solution concepts in order to refine our predictions.